Measurement of the group delay of laser mirrors by a Fabry–Perot interferometer

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The group delay of multilayer laser mirrors was measured by determining the spectral position of transmission maxima of a Fabry–Perot interferometer formed by the mirrors to be characterized. By optimizing the spacer thickness, we obtained an accuracy of the group-delay measurement of better than 0.23 fs. To our knowledge, this is the most precise direct measuring method reported.

As became known in the early days of femtosecond lasers[1,2] and was analyzed recently,[3] the group-delay dispersion (GDD) of multilayer mirrors has an effect on the duration and the phase structure of ultrashort pulses. On one hand, this means that proper care should be taken when one selects multilayer laser mirrors for intracavity applications and for beam-steering purposes. On the other hand, specially designed high-reflectivity laser mirrors have proved to be good candidates for controlling both intracavity and extracavity GDD in femtosecond lasers.[4–7] The recently generated transform-limited pulse in a mirror-dispersion-controlled Ti:sapphire laser is as short as 8 fs.[8] To construct such a high-performance laser, it is essential to have precise knowledge of the expected phase shift versus frequency.[9–12] In this Letter we present a very accurate, simple, and fairly inexpensive experimental technique for measuring the group delay of multilayer laser mirrors over a wide spectral range.

Let us take a dielectric mirror that shifts the phase of the reflected light by $\phi(\omega)$. If such mirrors form part of a Fabry–Perot interferometer (FPI) with a vacuum spacer of thickness h, then the condition of maximum transmission at a certain frequency depends on $\phi(\omega)$ and is written in general form as

$$\delta(\omega) = 2\left[\frac{h \cos \alpha}{c}\,\omega + \phi(\omega)\right] = 2\pi i\,, \qquad (1)$$

where i is the order of interference, α is the angle of incidence, and c is the velocity of light in vacuum. Thus from the spectral positions of neighboring transmission maxima the phase shift and the group delay τ of the mirror can uniquely be derived as

$$\tau(\omega_{i,i-1}) = \frac{\mathrm{d}\phi}{\mathrm{d}\omega}\bigg|_{\omega_{i,i-1}} \approx \frac{\phi(\omega_i) - \phi(\omega_{i-1})}{\omega_i - \omega_{i-1}} = -\frac{h \cos \alpha}{c} + \frac{\lambda_i \lambda_{i-1}}{2c(\lambda_i - \lambda_{i-1})}\,, \qquad (2)$$

where $\omega_{i,i-1} = (\omega_i + \omega_{i-1})/2$. In practice usually the wavelength is measured, so the quantities derived in frequency are expressed in wavelength.

Because this method is based on multiple-beam interference rather than on two-beam interference,[12] it is expected to be more accurate the sharper the resonance is. In practice the resonance sharpness of a FPI, however, is affected by the possible fluctuation of the spacer thickness that is due to slight thermal and mechanical instability. An experimental resolution of this problem is that the whole spectrum of interest is recorded at the same time within a few seconds.

The precision achievable with a FPI is therefore determined by the error $\Delta\lambda$ in the wavelength measurement. This $\Delta\lambda$ can be thought of as the exact position of a transmission maximum not measurable above a certain intensity level T_L (Fig. 1). From the well-known formula

$$T(\omega) = \frac{(1 - R)^2}{(1 - R)^2 + 4R \sin^2[\delta(\omega)/2]} \qquad (3)$$

of the transmission of a FPI consisting of two identical mirrors with reflectivity R, besides the frequency, we find that $\Delta\lambda$ is affected by R and the spacer thickness h in $\delta(\omega)$ [Eq. (1)]. The effect of R on $\Delta\lambda$ is quite clear: the method works better if mirrors with higher reflectivity are to be characterized. Throughout this Letter we assume that the reflectivities of two neighboring maxima are just slightly different from each other.

The effect of h, however, is more complicated. On one hand, the increase of the spacer thickness results in a smaller free spectral range. Consequently the numerical error of group delay $\Delta\tau_n$, which originates from approximating the tangent of $\phi(\omega)$ at $(\omega_i + \omega_{i-1})/2$ by the difference between $\phi(\omega_i)$ and $\phi(\omega_{i-1})$ in relation (2), defined as

$$\Delta\tau_n = \frac{\mathrm{d}\phi(\omega)}{\mathrm{d}\omega}\bigg|_{\omega_{i,i-1}} - \frac{\phi(\omega_i) - \phi(\omega_{i-1})}{\omega_i - \omega_{i-1}}\,, \qquad (4)$$

is smaller. Taking the Taylor series of $\phi(\omega)$ up to third order, we can write this error in the form

$$\Delta\tau_n = \frac{(\pi - \Delta\phi)^2}{24}\,\frac{\mathrm{d}^3\phi(\omega)}{\mathrm{d}\omega^3}\bigg|_{\omega_{i,i-1}} \left(\frac{c}{h'}\right)^2, \qquad (5)$$

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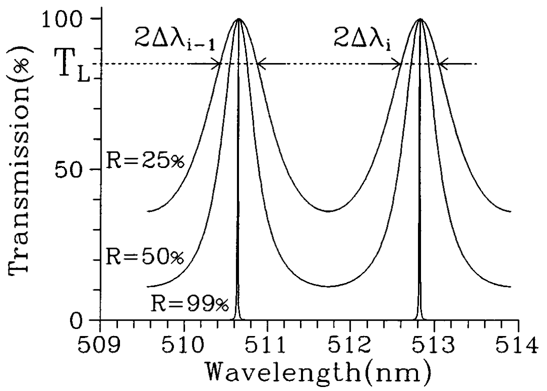

Fig. 1. Transmission of a FPI at different reflectivities.

where $\Delta\phi = \phi(\omega_i) - \phi(\omega_{i-1})$ and $h' = h \cos \alpha$. On the other hand, in relation (2) for the group delay the uncertainty $\Delta\lambda$ in measurement of the wavelength determined by R and h results in an error:

$$\Delta\tau_m = \frac{1}{2c} \frac{(\lambda_i + \lambda_{i-1})(\lambda_{i-1} - \lambda_i) - 2\lambda_i\lambda_{i-1}}{(\lambda_{i-1} - \lambda_i)^2} \Delta\lambda, \quad (6)$$

where $\Delta\lambda_{i-1} \cong \Delta\lambda_i = \Delta\lambda$ and $\Delta(\lambda_{i-1} - \lambda_i) = 2\Delta\lambda$ are assumed. The latter presumes the most pessimistic case, that is, the errors made in measuring the two neighboring maxima have different signs. Larger spacer thickness leads to narrower transmission maxima; hence the full width of the T_L level ($2\Delta\lambda$) is smaller. The error of measurement $\Delta\tau_m$ can eventually be expressed as

$$\Delta\tau_m \cong \frac{2\pi}{(\pi - \Delta\phi)^2} \sqrt{\frac{1 - T_L}{T_L F}} \frac{h'}{c}, \quad (7)$$

where $F = 4R/(1 - R)^2$. Since $\Delta\tau_n \sim h^{-2}$ and $\Delta\tau_m \sim h$, one can find a spacer thickness at which the uncertainty in determination of the group delay is minimum. This optimum spacer value is dependent on the given mirror as well as on the spectral range.

In accordance with our introductory remarks, two types of mirror have been measured. One is a low-dispersion dielectric sample beam-steering ($\alpha = 0$) mirror of an Ar^+ laser, which has reflectivity $R \geq 99.5\%$ and zero GDD between 480 and 530 nm. A reasonable phase shift can be expected only where the reflection [Fig. 2(a)] changes significantly.[13] The other is an intracavity mirror of an optical parametric oscillator (OPO), which has been designed to have $R \geq 99.5\%$ and approximately $-85\ fs^2$ GDD between 1100 and 1400 nm.[6,7] Both dielectric structures, consisting of TiO_2 and SiO_2 layers, have been evaporated onto BK7 substrates ($\lambda/10$ for the whole surface) by the Optical Coating Laboratory of the Research Institute for Solid State Physics, Budapest, Hungary. The absorption is negligible within the spectral ranges investigated.

In the first experiment we used a simple experimental apparatus available in most laboratories dealing with lasers and spectroscopy. A pointlike white-light source (a halogen lamp) was imaged onto the entrance slit of a high-resolution spectrograph (DFS-8, 0.6 nm/mm) by two achromatic lenses. The FPI consisting of two identical mirrors was inserted between the lenses ($\alpha = 0$). The positions of transmission maxima recorded on a photographic sheet film were evaluated by a Zeiss-made comparator. The typical readout and the spectral accuracy are $\pm 4\ \mu m$ ($80\ \mu m$) and $\Delta\lambda = \pm 0.0024$ nm (0.05 nm), respectively. The numbers in parentheses give the pessimistic estimation of greater error that is due to broader spectral stripes at the lower finesse range.

The approximate value of h was established from the wavelengths of transmission maxima in the regime where zero GDD was expected, i.e., where the reflection was not below 99%. The average spacer thickness was found to be $61.143 \pm 0.198\ \mu m$ within 480–530 nm.

Figure 2(b) shows the measured and designed values of group delay of the Ar^+ sample mirror. Significant deviations from the constant group-delay value are at approximately 400 and 440 nm, corresponding to steep changes of reflection coefficient [see Fig. 2(a)]. As one can see, the agreement between the measured and designed values is excellent. Since the shape of a femtosecond laser pulse is determined by GDD, the constant error of group delay, which is due to the inaccuracy in the determination of spacer thickness, plays no role in practice. In accordance with the theoretical prediction, the measured and averaged readout accuracy of ± 0.0024 nm, which has been determined from three measurement runs, results in 0.23-fs uncertainty in the determination of group delay in the high-reflection range.

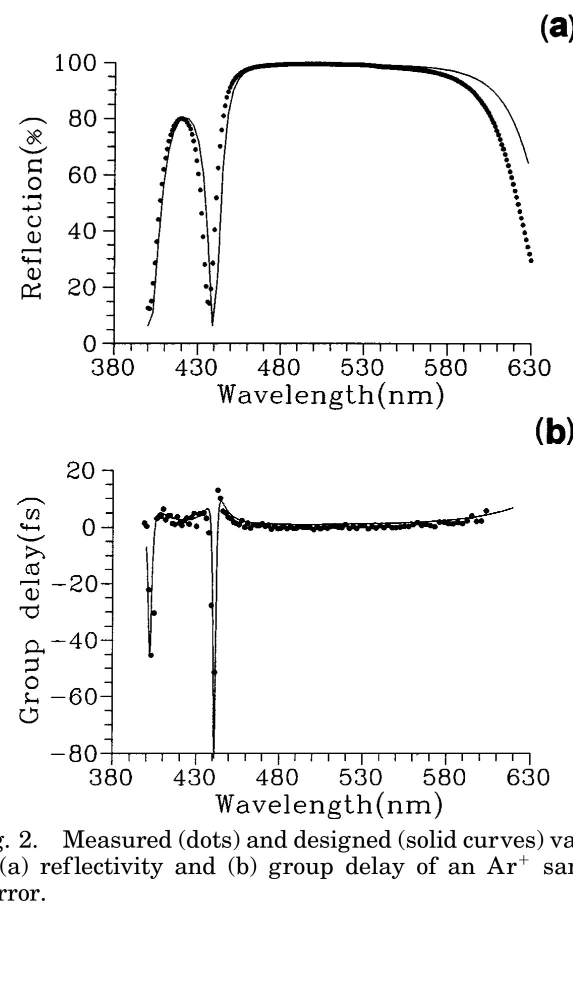

Fig. 2. Measured (dots) and designed (solid curves) values of (a) reflectivity and (b) group delay of an Ar^+ sample mirror.

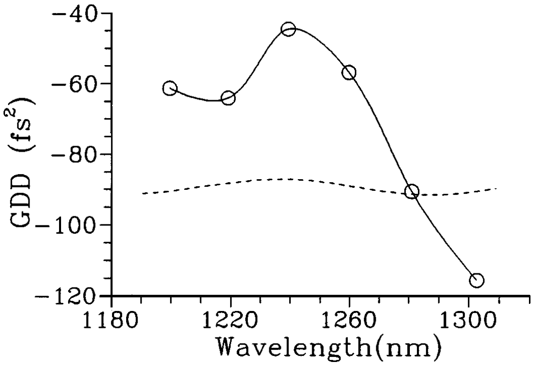

Fig. 3. Measured (circles) and designed (dashed curve) GDD of an intracavity GDD-controlling OPO mirror.

In some cases when the amount of detectable light is critical the FPI can be built from one high reflector to be characterized and a semitransparent mirror. This configuration, which is used in the reflection mode, resembles a Gires–Tournois interferometer. An ideal Gires–Tournois interferometer is expected to have flat power reflectance, but a real one that has absorption exhibits resonance minima in the spectrum of the reflected beam. Thus in the second experiment the interferometer was constructed from an OPO mirror ($R \approx 100\%$), and a gold mirror ($R \approx 80\%$) proved to have zero GDD.[12] The interferometer was illuminated from the gold mirror side at a small angle ($\alpha \approx 5°$), and the reflected light was then imaged onto the entrance slit of an HP70950A scanning optical spectrum analyzer.

It is clear that the precise value of spacer thickness cannot be determined where a significant phase shift is expected [see Eq. (1)]. In this second experiment, moreover, recording of the whole wavelength range with the required spectral resolution was not possible because of the limited number of diode channels of the spectrograph. The spectrum therefore was reconstructed from subsequent scans whose recording took a few minutes, so even a rough value of h could not be established because of the fluctuating spacer thickness. These difficulties in the determination of h can be eliminated by noticing that the GDD can be expressed directly from relation (2) as

$$\mathrm{GDD} = \frac{\mathrm{d}\tau(\omega)}{\mathrm{d}\omega}\bigg|_{\omega_i} \approx \frac{\tau(\omega_{i+1,i}) - \tau(\omega_{i,i-1})}{\omega_{i+1,i} - \omega_{i,i-1}} = \frac{1}{2\pi c^2} \times \left(\frac{\lambda_{i+1}\lambda_{i-1}}{\lambda_{i-1} - \lambda_{i+1}}\right)\left(\frac{\lambda_{i+1}\lambda_i}{\lambda_{i+1} - \lambda_i} - \frac{\lambda_i\lambda_{i-1}}{\lambda_i - \lambda_{i-1}}\right). \quad (8)$$

Accordingly, the measurement of the chirped OPO mirror was carried out by scan-to-scan recording of the required spectral range of 1160–1320 nm. Each scan had three resonances. Figure 3 shows the GDD calculated from the measured wavelength of the reflection minima for each scan. In this experiment the uncertainty of GDD measurement was found to be $\pm 5\ \mathrm{fs}^2$. The disagreement between the designed (dashed curve) and measured (circles) values, which has been independently verified by measurement of the pulse length of the OPO,[7] is the result of this OPO mirror's being the first trial sample. This also supports the need for a device by which the control of multilayer mirrors having complicated structure can be made promptly after fabrication.

It is important to point out that in both cases the extremely high accuracy has been achieved by direct evaluation of the measured data, that is, wihout using any fitting algorithm to a presumed (polynomial) function of group delay. This may be of special interest in practical cases for which the fitting of group delay could not be fast and easy [e.g., see Fig. 2(b)] or the spacer thickness could not be determined precisely.

We have introduced a new technique for measuring the group delay of multilayer laser mirrors, based on such simple spectroscopic devices as a Fabry–Perot interferometer and a spectrograph. The method, which does not involve a femtosecond laser, is capable of giving the values of group delay with 0.23-fs accuracy directly, which to our knowledge has never been achieved before without the use of serious fitting procedures. The accuracy has been studied theoretically as well. It is pointed out that at a given finesse value of the interferometer the error can be minimized by appropriate choice of the spacer thickness.

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